Summary

Considerations of computer memory and time in the numerical solution of differential equations defined over a spatial domain of infinite or very large size leads one to truncating the domain under consideration to a smaller (interior) region of interest. In such a situation, the question of the condition to be imposed at the truncation boundary becomes important, especially when the phenomenon under consideration is that of wave propagation such that the waves are outgoing. A wrong boundary condition (b.c.) in such a case physically amounts to nonhomogeneity in the medium; computationally it leads to spurious reflection of waves into the interior domain. In order to model infinite or very large domains by smaller finite computational domains accurately for wave propagation problems, Non Reflecting Boundary Conditions (NRBC) must be imposed at the truncation boundary. NRBC's do not allow reflection of waves originating in the interior region and striking the truncation boundary back into the interior, thus modeling the actual physics. Modeling of unbounded media is an active field of research, its aim being to solve practical problems efficiently and accurately. The region of interest in the spatial domain is the *near* (interior) region, the *far* (exterior) region excluded is generally assumed to be regular. An NRBC replaces the combination of the infinite domain and a radiation condition generally applicable at infinity for the problem posed over the unbounded domain. This dissertation deals with problems in the time domain, and the theme is modeling of unbounded media using NRBC's.

The exact form of an NRBC is, in general, *global in space and time*. Computational effort can be reduced by constructing a simple but, in general, an approximate form in place of the above. In the time domain, this simpler but approximate relationship is usually *local in space and time*.

Chapter 1 covers an introduction to NRBC's, to the situations in which they arise, and the advantage of using them. The methodology of NRBC's for solving wave

fields over unbounded/ large domains is placed in context with relation to field problems posed over unbounded domains in general, and also in relation to methods other than the procedure of NRBC's that have been employed for solving wave propagation problems in unbounded media. The scope of the current study is outlined, and an overview of the work presented in the remainder of the dissertaion is provided.

Chapter 2 provides a literature review pertaining to NRBC's and also deals with other methods for modeling infinite media for wave problems. The fact that NRBC's express a large variety in their conceptual basis, mathematical formulation, and numerical implementation is highlighted. Depending on the properties of the unbounded medium, the approach towards formulating an NRBC as well as the NRBC itself are different. An NRBC generally is an approximation, and there can be many approximations to the same situation: for the same unbounded medium (the same governing differential equation), different approaches towards formulating an NRBC will result in different NRBC's. The efficacy of a particular NRBC can be judged on the basis of how much of the spurious reflection it is able to suppress, or similar other criteria.

The above observations are highlighted in the problems considered and the work reported by the author in this field. This work consists of formulation and application of NRBC's for two different classes of problems. The first involves the time dependent wave equation in two dimensions for a *complex* geometry. The second deals with transonic flow of a Newtonian fluid over a flat plate. The first one is a second order linear partial differential equation (p.d.e.) with constant coefficients, the second one consists of *compressible* Navier Stokes equations which comprises a nonlinear set of p.d.e.'s. Chapters three and four report the work carried out on these two problems, respectively. In order to carry out the numerical experiments, computer programs for the respective problems were developed from scratch.

The third chapter deals with the formulation and well posedness analysis of a particular class of NRBC's for the time dependent scalar wave equation. These NRBC's are then applied to a problem of great importance– to that of large heat exchangers in nuclear and process industries. A methodolgy is developed to compute the interaction of failure induced pressure waves emanating first from a burst high pressure tube, then spreading through a low pressure liquid, and reflecting from the neighboring pressure tubes. The effect of this pressure field on the tubes in the *immediate* neighbourhood of the tube that fails needs to be investigated to determine their likelihood of failure in turn. The proposed method affords significant economic improvement for computation by reducing the spatial domain of computation through the use of truncating boundaries where appropriate NRBC's are applied. These NRBC's simulate the behaviour of the large expanse of the low pressure fluid outside the computational domain. Perfectly reflecting boundary is assumed for the multiple tubes in the domain. Well posedness analysis of the IBVP obtained when the problem is posed with these new set of boundary conditions applicable at the artificial boundary introduced at a finite distance from the source(s) and scatterer(s), rather than the original IBVP posed in terms of the Sommerfeld radiation condition which is applicable only at large distances from the sources/scatterers, is presented. Previous work reported in the literature has used Sommerfeld radiation condition for this problem at the truncated moderator domain. It is demonstrated that the use of Sommerfeld radiation condition at finite distances in the spatial domain results in spurious reflections. Computational results obtained for a simplified domain are compared with the corresponding analytical solution to verify the applicability of the method before using the method for the complex geometry of the PHWR. An elaborate computer experimentation is carried out on the placement of truncating boundaries. Transient phase of the solution is concentrated upon. The governing differential equation used is the scalar wave equation. The problem is linear.

In the fourth chapter, three *new* NRBC's for the compressible Navier Stokes equations are formulated. The new NRBC's proposed are characterised by two tunable parameters, and by the way they handle the term containing the pressure at the subsonic outflow boundary. In order to investigate the performance of these NRBC's, they are applied to a simple geometry of a semi infinite flat plate placed in a uniform transonic stream of air. Steady state solution is then calculated using a time marching technique for this problem. Numerical experimentation is carried out to study the effect of the NRBC applied and of the two parameters on the number of steps required for converging to the steady state. The criteria of minimum spurious reflections from the subsonic outflow boundary is related to the minimum steps required for convergence to steady state. Spurious reflections are graphically observed by plotting pressure history at a point near the subsonic outflow boundary, oscillations in this graph are indeed reduced when convergence to steady state is accelerated. Setting one of the tunable parameters to 1 in one of the newly proposed NRBC's results in an NRBC already reported in literature. It is seen from numerical experiments that this special choice need not result in minimizing the steps required for convergence. The boundary layer at steady state is captured in the steady state solution. We deal with a non linear set of governing differential equations and boundary conditions but a simple geometry in this part of the work. Results of numerical experimentation with these NRBC's has been presented, which can serve as a guideline for the choice of the parameters for more complicated flowfields.

The fifth chapter contains the main conclusions arrived at from the study. A critical evaluation of the work is presented and further scope of study in this area outlined in this last chapter.